Oxford Cambridge and RSA

Monday 4 October 2021 – Afternoon

AS Level Further Mathematics A

Y531/01 Pure Core

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 The lines l_1 and l_2 have the following equations.

$$l_1 : \mathbf{r} = \begin{pmatrix} 8 \\ -11 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$$
$$l_2 : \mathbf{r} = \begin{pmatrix} -6 \\ 11 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix}$$

(a) Show that l_1 and l_2 intersect. **[4]**

(b) Write down the point of intersection of l_1 and l_2. **[1]**

2 The equation $2x^3 + 3x^2 - 2x + 5 = 0$ has roots α, β and γ.

Use a substitution to find a cubic equation with integer coefficients whose roots are $\alpha + 1$, $\beta + 1$ and $\gamma + 1$. **[4]**

3 **In this question you must show detailed reasoning.**

The equation $x^4 - 7x^3 - 2x^2 + 218x - 1428 = 0$ has a root $3 - 5\mathrm{i}$.

Find the other three roots of this equation. **[6]**

4 **(a)** A locus C_1 is defined by $C_1 = \{z : |z + \mathrm{i}| \leqslant |z - 2|\}$.

(i) Indicate by shading on the Argand diagram in the Printed Answer Booklet the region representing C_1. **[2]**

(ii) Find the cartesian equation of the boundary line of the region representing C_1, giving your answer in the form $ax + by + c = 0$. **[2]**

(b) A locus C_2 is defined by $C_2 = \{z : |z + 1| \leqslant 3\} \cap \{z : |z - 2\mathrm{i}| \geqslant 2\}$.

Indicate by shading on the Argand diagram in the Printed Answer Booklet the region representing C_2. **[3]**

5 Matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} \frac{5}{13} & -\frac{12}{13} \\ \frac{12}{13} & \frac{5}{13} \end{pmatrix}$.

(a) Use **A** and **B** to disprove the proposition: "Matrix multiplication is commutative". **[2]**

Matrix **B** represents the transformation $\mathrm{T_B}$.

(b) Describe the transformation $\mathrm{T_B}$. **[2]**

(c) By considering the inverse transformation of $\mathrm{T_B}$, determine $\mathbf{B}^{-1}$. **[2]**

Matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -3 \end{pmatrix}$ and represents the transformation $\mathrm{T_C}$.

The transformation $\mathrm{T_{BC}}$ is transformation $\mathrm{T_C}$ followed by transformation $\mathrm{T_B}$.

An object shape of area 5 is transformed by $\mathrm{T_{BC}}$ to an image shape N.

(d) Determine the area of N. **[2]**

6 **In this question you must show detailed reasoning.**

(a) Solve the equation $2z^2 - 10z + 25 = 0$ giving your answers in the form $a + b\mathrm{i}$. **[2]**

(b) Solve the equation $3\omega - 2 = \mathrm{i}(5 + 2\omega)$ giving your answer in the form $a + b\mathrm{i}$. **[4]**

7 Prove that $2^{3n} - 3^n$ is divisible by 5 for all integers $n \geqslant 1$. **[5]**

8 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} t-1 & t-1 & t-1 \\ 1-t & 6 & t \\ 2-2t & 2-2t & 1 \end{pmatrix}$.

(a) Find, in fully factorised form, an expression for det **A** in terms of t. **[3]**

(b) State the values of t for which **A** is singular. **[1]**

You are given the following system of equations in x, y and z, where b is a real number.

$$\begin{aligned} (b^2+1)x + (b^2+1)y + (b^2+1)z &= 5 \\ (-b^2-1)x + 6y + (b^2+2)z &= 10 \\ (-2b^2-2)x + (-2b^2-2)y + z &= 15 \end{aligned}$$

(c) Determine which **one** of the following statements about the solution of the equations is true.

- There is a unique solution for all values of b.
- There is a unique solution for some, but not all, values of b.
- There is no unique solution for any value of b. **[2]**

9 The points $P(3, 5, -21)$ and $Q(-1, 3, -16)$ are on the ceiling of a long straight underground tunnel. A ventilation shaft must be dug from the point M on the ceiling of the tunnel midway between P and Q to horizontal ground level (where the z-coordinate is 0). The ventilation shaft must be perpendicular to the tunnel.

The path of the ventilation shaft is modelled by the vector equation $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where $\mathbf{a}$ is the position vector of M.

You are given that $\mathbf{b} = \begin{pmatrix} 1 \\ s \\ t \end{pmatrix}$ where s and t are real numbers.

(a) Show that $s = 2.5t - 2$. **[3]**

(b) Show that at the point where the ventilation shaft reaches the ground $\lambda = \frac{c}{t}$, where c is a constant to be determined. **[3]**

(c) Using the results in parts **(a)** and **(b)**, determine the shortest possible length of the ventilation shaft. **[6]**

(d) Explain what the fact that $\mathbf{b} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \neq \mathbf{0}$ means about the direction of the ventilation shaft. **[1]**

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