



Oxford Cambridge and RSA

**Monday 4 October 2021 – Afternoon**

**AS Level Further Mathematics A**

**Y531/01 Pure Core**

**Time allowed: 1 hour 15 minutes**



**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- This document has **8** pages.

**ADVICE**

- Read each question carefully before you start your answer.

Answer **all** the questions.

- 1 The lines  $l_1$  and  $l_2$  have the following equations.

$$l_1 : \mathbf{r} = \begin{pmatrix} 8 \\ -11 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$$

$$l_2 : \mathbf{r} = \begin{pmatrix} -6 \\ 11 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix}$$

- (a) Show that  $l_1$  and  $l_2$  intersect. [4]

- (b) Write down the point of intersection of  $l_1$  and  $l_2$ . [1]

- 2 The equation  $2x^3 + 3x^2 - 2x + 5 = 0$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ .

Use a substitution to find a cubic equation with integer coefficients whose roots are  $\alpha + 1$ ,  $\beta + 1$  and  $\gamma + 1$ . [4]

- 3 **In this question you must show detailed reasoning.**

The equation  $x^4 - 7x^3 - 2x^2 + 218x - 1428 = 0$  has a root  $3 - 5i$ .

Find the other three roots of this equation. [6]

- 4 (a) A locus  $C_1$  is defined by  $C_1 = \{z : |z + i| \leq |z - 2|\}$ .

- (i) Indicate by shading on the Argand diagram in the Printed Answer Booklet the region representing  $C_1$ . [2]

- (ii) Find the cartesian equation of the boundary line of the region representing  $C_1$ , giving your answer in the form  $ax + by + c = 0$ . [2]

- (b) A locus  $C_2$  is defined by  $C_2 = \{z : |z + 1| \leq 3\} \cap \{z : |z - 2i| \geq 2\}$ .

Indicate by shading on the Argand diagram in the Printed Answer Booklet the region representing  $C_2$ . [3]

5 Matrices  $\mathbf{A}$  and  $\mathbf{B}$  are given by  $\mathbf{A} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} \frac{5}{13} & -\frac{12}{13} \\ \frac{12}{13} & \frac{5}{13} \end{pmatrix}$ .

- (a) Use  $\mathbf{A}$  and  $\mathbf{B}$  to disprove the proposition: “Matrix multiplication is commutative”. [2]

Matrix  $\mathbf{B}$  represents the transformation  $T_{\mathbf{B}}$ .

- (b) Describe the transformation  $T_{\mathbf{B}}$ . [2]

- (c) By considering the inverse transformation of  $T_{\mathbf{B}}$ , determine  $\mathbf{B}^{-1}$ . [2]

Matrix  $\mathbf{C}$  is given by  $\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -3 \end{pmatrix}$  and represents the transformation  $T_{\mathbf{C}}$ .

The transformation  $T_{\mathbf{BC}}$  is transformation  $T_{\mathbf{C}}$  followed by transformation  $T_{\mathbf{B}}$ .

An object shape of area 5 is transformed by  $T_{\mathbf{BC}}$  to an image shape  $N$ .

- (d) Determine the area of  $N$ . [2]

6 In this question you must show detailed reasoning.

- (a) Solve the equation  $2z^2 - 10z + 25 = 0$  giving your answers in the form  $a + bi$ . [2]

- (b) Solve the equation  $3\omega - 2 = i(5 + 2\omega)$  giving your answer in the form  $a + bi$ . [4]

- 7 Prove that  $2^{3n} - 3^n$  is divisible by 5 for all integers  $n \geq 1$ . [5]

4

8 The matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} t-1 & t-1 & t-1 \\ 1-t & 6 & t \\ 2-2t & 2-2t & 1 \end{pmatrix}$ .

(a) Find, in fully factorised form, an expression for  $\det \mathbf{A}$  in terms of  $t$ . [3]

(b) State the values of  $t$  for which  $\mathbf{A}$  is singular. [1]

You are given the following system of equations in  $x$ ,  $y$  and  $z$ , where  $b$  is a real number.

$$\begin{aligned} (b^2 + 1)x + (b^2 + 1)y + (b^2 + 1)z &= 5 \\ (-b^2 - 1)x + 6y + (b^2 + 2)z &= 10 \\ (-2b^2 - 2)x + (-2b^2 - 2)y + z &= 15 \end{aligned}$$

(c) Determine which **one** of the following statements about the solution of the equations is true.

- There is a unique solution for all values of  $b$ .
- There is a unique solution for some, but not all, values of  $b$ .
- There is no unique solution for any value of  $b$ .

[2]

- 9 The points  $P(3, 5, -21)$  and  $Q(-1, 3, -16)$  are on the ceiling of a long straight underground tunnel. A ventilation shaft must be dug from the point  $M$  on the ceiling of the tunnel midway between  $P$  and  $Q$  to horizontal ground level (where the  $z$ -coordinate is 0). The ventilation shaft must be perpendicular to the tunnel.

The path of the ventilation shaft is modelled by the vector equation  $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ , where  $\mathbf{a}$  is the position vector of  $M$ .

You are given that  $\mathbf{b} = \begin{pmatrix} 1 \\ s \\ t \end{pmatrix}$  where  $s$  and  $t$  are real numbers.

- (a) Show that  $s = 2.5t - 2$ . [3]
- (b) Show that at the point where the ventilation shaft reaches the ground  $\lambda = \frac{c}{t}$ , where  $c$  is a constant to be determined. [3]
- (c) Using the results in parts (a) and (b), determine the shortest possible length of the ventilation shaft. [6]
- (d) Explain what the fact that  $\mathbf{b} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \neq \mathbf{0}$  means about the direction of the ventilation shaft. [1]

**END OF QUESTION PAPER**

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